Flavour models and CPV

Michael A. Schmidt

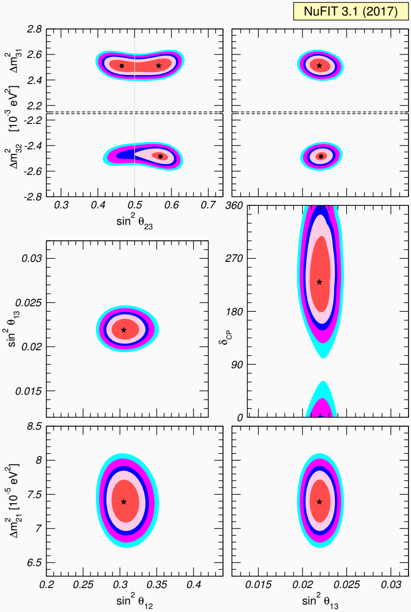
21 December 2017

NuPhys 2017

THE UNIVERSITY OF SYDNEY

CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

Current status: global fit to neutrino oscillation experiments



Leptonic mixing (PMNS) matrix

$$U_{PMNS} = R_{23}(\theta_{23})U_{13}(\theta_{13},\delta)R_{12}(\theta_{12})$$

with rotation matrices such as

$$U_{13}(\theta_{13},\delta) = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

	any ordering	close to
$\sin^2\theta_{12}$	$0.272 \to 0.347$	$\frac{1}{3}$
$\sin^2\theta_{23}$	$0.401 \to 0.628$	$\frac{1}{2}$
$\sin^2\theta_{13}$	$0.01971 \to 0.02434$	≈ 0
$\delta[^\circ]$	$128 \to 390$	

Symmetries work well for gauge interact's.

Is there a symmetry in the lepton sector?

Outline

Goal: predict (mixing) parameters in lepton sector

1. Flavour symmetry
2. Flavour symmetry + CP
3. Symmetry breaking

Considerable effort in flavour model building:
Altarelli, Antusch, Babu, Ballett, Branco, Cai, Centelles Chulia, Chen, Chu, Dasgupta, de Medeiros Varzielas, Di Iura, Ding, Everett, Fallbacher, Feruglio, Frampton, Ge, Gehrlein, Girardi, Gonzalez Felipe, Grimus, Hagedorn, He, Hernandez, Holthausen, Joaquim, Keum, King, Lam, Lavoura, Lindner, Ludl, Luhn, Ma, Mahanthappa, Machado, Meloni, Meroni, Mohapatra, Molinaro, Neder, Nishi, Päs, Pascoli, Penedo, Petcov, Ramond, Ratz, Rodejohann, Schumacher, Serodio, Shimizu, Smirnov, Spinrath, Srivastava, Stuart, MS, Tanimoto, Titov, Trautner, Turner, Valle, Vien, Volkas, Xu, Yamamoto, Yu, Ziegler, Zhou, ...

Posters at νPhys:
João Penedo *"Neutrino Mixing and Leptonic CP Violation from S4 Flavour and Generalised CP Symmetries"*
Arsenii Titov *"Probing Neutrino Mixing Schemes in DUNE and T2HK"*
Ye-Ling Zhou *"Effect vacuum alignments as building blocks of flavour models"*

See recent reviews by Steve King 1510.02091 and Serguey Petcov 1711.10806

Outline

Goal: predict (mixing) parameters in lepton sector

1. Flavour symmetry
2. Flavour symmetry + CP
3. Symmetry breaking

Considerable effort in flavour model building:
Altarelli, Antusch, Babu, Ballett, Branco, Cai, Centelles Chulia, Chen, Chu, Dasgupta, de Medeiros Varzielas, Di Iura, Ding, Everett, Fallbacher, Feruglio, Frampton, Ge, Gehrlein, Girardi, Gonzalez Felipe, Grimus, Hagedorn, He, Hernandez, Holthausen, Joaquim, Keum, King, Lam, Lavoura, Lindner, Ludl, Luhn, Ma, Mahanthappa, Machado, Meloni, Meroni, Mohapatra, Molinaro, Neder, Nishi, Päs, Pascoli, Penedo, Petcov, Ramond, Ratz, Rodejohann, Schumacher, Serodio, Shimizu, Smirnov, Spinrath, Srivastava, Stuart, MS, Tanimoto, Titov, Trautner, Turner, Valle, Vien, Volkas, Xu, Yamamoto, Yu, Ziegler, Zhou, ...

Posters at νPhys:
João Penedo *"Neutrino Mixing and Leptonic CP Violation from S4 Flavour and Generalised CP Symmetries"*
Arsenii Titov *"Probing Neutrino Mixing Schemes in DUNE and T2HK"*
Ye-Ling Zhou *"Effect vacuum alignments as building blocks of flavour models"*

See recent reviews by Steve King 1510.02091 and Serguey Petcov 1711.10806

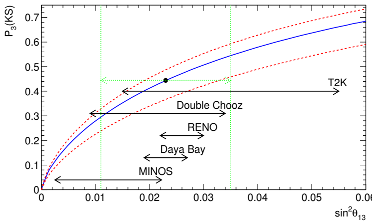
Benchmark: anarchy

- All mixing parameters are random variables
  Hall, Murayama, Weiner hep-ph/9911341; Haba, Murayama hep-ph/0009174; deGouvea, Murayama hep-ph/0301050, 1204.1249
- Probability distribution invariant under change of basis
- Flat in s_{12}^2, s_{23}^2, c_{13}^4 and phases due to Haar measure and mutually independent in these variables

- Kolmogorov-Smirnov test
  Probability that anarchy hypothesis is consistent with experimental data hep-ph/0301050

$$P_3^{KS} = \epsilon_3 \left(1 - \ln \epsilon_3 + \frac{1}{2} \ln^2 \epsilon_3\right)$$

$$\epsilon_3 = 2\, s_{12}^2 \times 2 \min(s_{23}^2, c_{23}^2) \times 2\,(1 - c_{13}^4)$$



1204.1249

- Expect large CP phase δ: distribution of $\sin\delta$ peaked at $\pm 1 \to \delta \simeq \pm\frac{\pi}{2}$
- However KS test requires many data points to reject null hypothesis
- Combination of (smaller) symmetry with anarchy

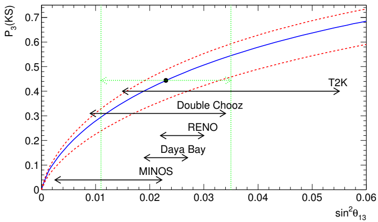
Antonelli,Caravaglios,Ferrari, Picariello hep-ph/0207347; Altarelli, Feruglio, Masina hep-ph/0210342; ...

Benchmark: anarchy

- All mixing parameters are random variables
  Hall, Murayama, Weiner hep-ph/9911341; Haba, Murayama hep-ph/0009174; deGouvea, Murayama hep-ph/0301050, 1204.1249
- Probability distribution invariant under change of basis
- Flat in s_{12}^2, s_{23}^2, c_{13}^4 and phases due to Haar measure and mutually independent in these variables
- Kolmogorov-Smirnov test
  Probability that anarchy hypothesis is consistent with experimental data hep-ph/0301050

$$P_3^{KS} = \epsilon_3 \left(1 - \ln \epsilon_3 + \frac{1}{2} \ln^2 \epsilon_3\right)$$

$$\epsilon_3 = 2\, s_{12}^2 \times 2 \min(s_{23}^2, c_{23}^2) \times 2\,(1 - c_{13}^4)$$



1204.1249

- Expect large CP phase δ: distribution of $\sin\delta$ peaked at $\pm 1 \rightarrow \delta \simeq \pm\frac{\pi}{2}$
- However KS test requires many data points to reject null hypothesis
- Combination of (smaller) symmetry with anarchy

Antonelli,Caravaglios,Ferrari, Picariello hep-ph/0207347; Altarelli, Feruglio, Masina hep-ph/0210342; ...

Mixing from flavour symmetries

Flavour symmetries

Properties of symmetry

- Continuous vs. discrete: because more group invariants
- Abelian vs. non-Abelian: in order to get correlations
- Subgroup of largest flavour group in SM $G_f \leq U(3)^5$

$\rightarrow$ usually discrete subgroup of $U(3)$ (or $SU(3)$)

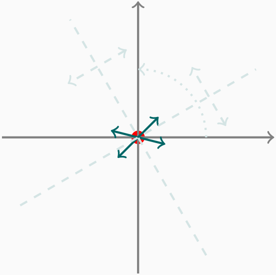
- with 3-dimensional representation [due to 3 generations]
- Flavour symmetry G_f must be broken at low energies: explicit vs. spontaneous

Mixing from symmetry breaking

Point symmetries of plane

- reflections about origin
- and lines through origin

$\rightarrow$ includes rotations SO(2)



Symmetry breaking
by oriented lines (vectors) in plane

Little group (remnant symmetry):

- reflection about vector

Misalignment angle θ between vectors determined by symmetry breaking

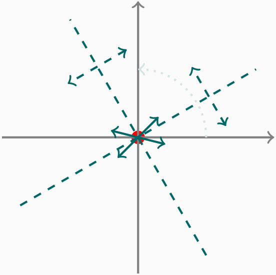
- Group of reflections $\rightarrow$ general group G
- Vector $\rightarrow$ representation of group G
- Remnant reflection $\rightarrow$ subgroup of G (including trivial subgroup $\{\mathbf{1}\}$)

Mixing from symmetry breaking

Point symmetries of plane

- reflections about origin
- and lines through origin

$\rightarrow$ includes rotations SO(2)



Symmetry breaking
by oriented lines (vectors) in plane

Little group (remnant symmetry):

- reflection about vector

Misalignment angle θ between vectors determined by symmetry breaking

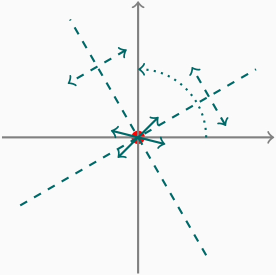
- Group of reflections $\rightarrow$ general group G
- Vector $\rightarrow$ representation of group G
- Remnant reflection $\rightarrow$ subgroup of G (including trivial subgroup $\{\mathbf{1}\}$)

Mixing from symmetry breaking

Point symmetries of plane

- reflections about origin
- and lines through origin

→ includes rotations SO(2)



Symmetry breaking
by oriented lines (vectors) in plane

Little group (remnant symmetry):

- reflection about vector

Misalignment angle θ between vectors determined by symmetry breaking

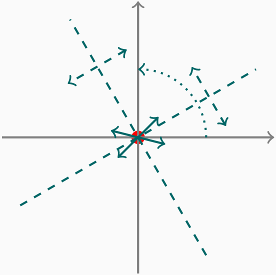
- Group of reflections → general group G
- Vector → representation of group G
- Remnant reflection → subgroup of G (including trivial subgroup $\{\mathbf{1}\}$)

Mixing from symmetry breaking

Point symmetries of plane

- reflections about origin
- and lines through origin

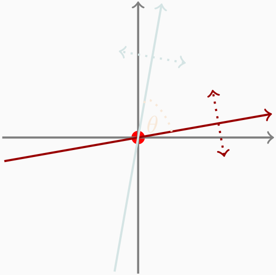
$\rightarrow$ includes rotations SO(2)



Symmetry breaking
by oriented lines (vectors) in plane

Little group (remnant symmetry):

- reflection about vector



Misalignment angle θ between vectors determined by symmetry breaking

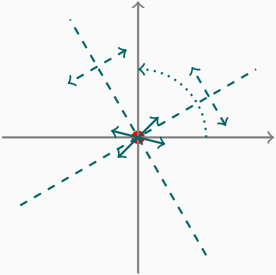
- Group of reflections $\rightarrow$ general group G
- Vector $\rightarrow$ representation of group G
- Remnant reflection $\rightarrow$ subgroup of G (including trivial subgroup $\{\mathbf{1}\}$)

Mixing from symmetry breaking

Point symmetries of plane

- reflections about origin
- and lines through origin

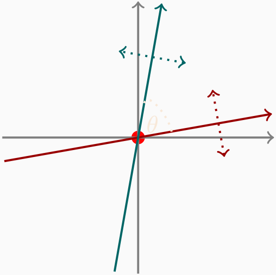
$\rightarrow$ includes rotations SO(2)



Symmetry breaking
by oriented lines (vectors) in plane

Little group (remnant symmetry):

- reflection about vector



Misalignment angle θ between vectors determined by symmetry breaking

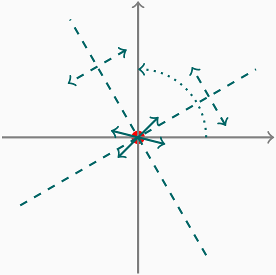
- Group of reflections $\rightarrow$ general group G
- Vector $\rightarrow$ representation of group G
- Remnant reflection $\rightarrow$ subgroup of G (including trivial subgroup $\{\mathbf{1}\}$)

Mixing from symmetry breaking

Point symmetries of plane

- reflections about origin
- and lines through origin

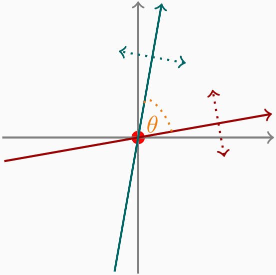
$\rightarrow$ includes rotations SO(2)



Symmetry breaking
by oriented lines (vectors) in plane

Little group (remnant symmetry):

- reflection about vector



Misalignment angle θ between vectors determined by symmetry breaking

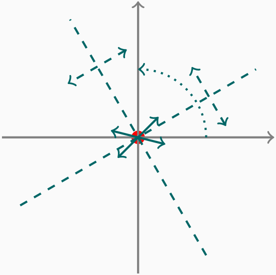
- Group of reflections $\rightarrow$ general group G
- Vector $\rightarrow$ representation of group G
- Remnant reflection $\rightarrow$ subgroup of G (including trivial subgroup $\{\mathbf{1}\}$)

Mixing from symmetry breaking

Point symmetries of plane

- reflections about origin
- and lines through origin

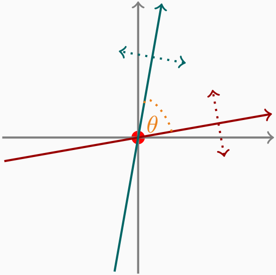
→ includes rotations SO(2)



Symmetry breaking
by oriented lines (vectors) in plane

Little group (remnant symmetry):

- reflection about vector



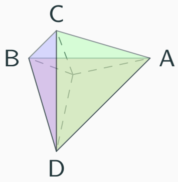
Misalignment angle θ between vectors determined by symmetry breaking

- Group of reflections → general group G
- Vector → representation of group G
- Remnant reflection → subgroup of G (including trivial subgroup $\{\mathbf{1}\}$)

Flavour symmetries: A_4 as example

$A_4 \cong$ symmetry of tetrahedron

[smallest group with 3-dimensional rep'n]



Predictions

- Mixing angles up to exchange of rows and columns
- Dirac phase δ up to π
- Majorana phases undetermined

Crucial ingredients

flavour symmetry and breaking pattern

$\rightarrow$ mixing is induced by misalignment

Mixing from flavour symmetry breaking

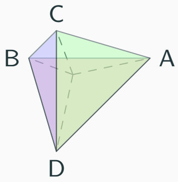
Altarelli, Feruglio hep-ph/0512103 He, Keum, Volkas hep-ph/0601001

- $U_\ell \rightarrow U_\ell K_\ell$ removes 3 unphysical phases
- $U_\nu \rightarrow U_\nu K_\nu$ for real positive ν masses
- Permutation of columns of $U_{\ell,\nu}$ possible $U_{\ell,\nu} \rightarrow U_{\ell,\nu} P_{\ell,\nu}$

Flavour symmetries: A_4 as example

$A_4 \cong$ symmetry of tetrahedron

[smallest group with 3-dimensional rep'n]



Predictions

- Mixing angles up to exchange of rows and columns
- Dirac phase δ up to π
- Majorana phases undetermined

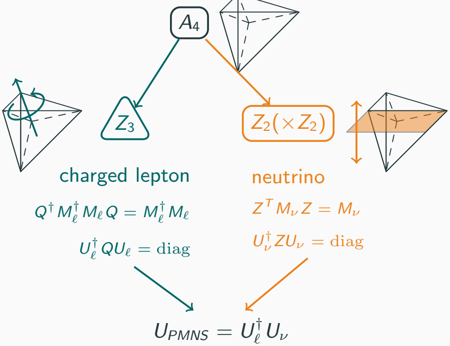
Crucial ingredients

flavour symmetry and breaking pattern

$\rightarrow$ mixing is induced by misalignment

Mixing from flavour symmetry breaking

Altarelli, Feruglio hep-ph/0512103 He, Keum, Volkas hep-ph/0601001

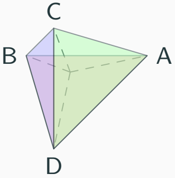


- $U_\ell \rightarrow U_\ell K_\ell$ removes 3 unphysical phases
- $U_\nu \rightarrow U_\nu K_\nu$ for real positive ν masses
- Permutation of columns of $U_{\ell,\nu}$ possible $U_{\ell,\nu} \rightarrow U_{\ell,\nu} P_{\ell,\nu}$

Flavour symmetries: A_4 as example

$A_4 \cong$ symmetry of tetrahedron

[smallest group with 3-dimensional rep'n]



Predictions

- Mixing angles up to exchange of rows and columns
- Dirac phase δ up to π
- Majorana phases undetermined

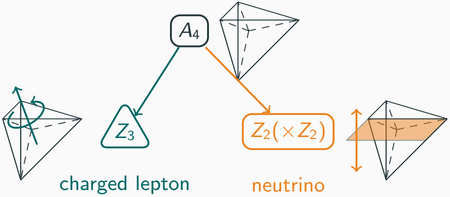
Crucial ingredients

flavour symmetry and breaking pattern

$\rightarrow$ mixing is induced by misalignment

Mixing from flavour symmetry breaking

Altarelli, Feruglio hep-ph/0512103 He, Keum, Volkas hep-ph/0601001



$$Q^\dagger M_\ell^\dagger M_\ell Q = M_\ell^\dagger M_\ell \qquad Z^T M_\nu Z = M_\nu$$

$$U_\ell^\dagger Q U_\ell = \mathrm{diag} \qquad U_\nu^\dagger Z U_\nu = \mathrm{diag}$$

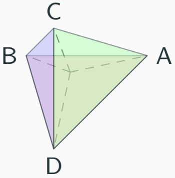
$$U_{PMNS} = U_\ell^\dagger U_\nu$$

- $U_\ell \rightarrow U_\ell K_\ell$ removes 3 unphysical phases
- $U_\nu \rightarrow U_\nu K_\nu$ for real positive ν masses
- Permutation of columns of $U_{\ell,\nu}$ possible
  $U_{\ell,\nu} \rightarrow U_{\ell,\nu} P_{\ell,\nu}$

Flavour symmetries: A_4 as example

$A_4 \cong$ symmetry of tetrahedron

[smallest group with 3-dimensional rep'n]



Predictions

- Mixing angles up to exchange of rows and columns
- Dirac phase δ up to π
- Majorana phases undetermined

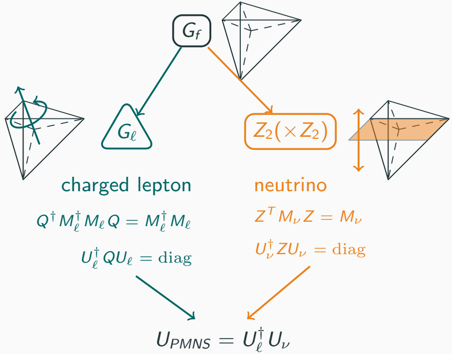
Crucial ingredients

flavour symmetry and breaking pattern

$\to$ mixing is induced by misalignment

Mixing from flavour symmetry breaking

Altarelli, Feruglio hep-ph/0512103 He, Keum, Volkas hep-ph/0601001



- $U_\ell \to U_\ell K_\ell$ removes 3 unphysical phases
- $U_\nu \to U_\nu K_\nu$ for real positive ν masses
- Permutation of columns of $U_{\ell,\nu}$ possible $U_{\ell,\nu} \to U_{\ell,\nu} P_{\ell,\nu}$

Popular mixing patterns

Bimaximal mixing

Fukugita,Tanimoto,Yanagida hep-ph/9709388

$$s_{13}^2 = 0,\ s_{12}^2 = s_{23}^2 = \frac{1}{2}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Prediction of S_4, ...

Tribimaximal mixing

Harrison,Perkins,Scott hep-ph/0202074

$$s_{13}^2 = 0,\ s_{12}^2 = \frac{1}{3},\ s_{23}^2 = \frac{1}{2}$$

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Prediction of A_4, S_4, ...

trimaximal mixing:
TM1: only first column
TM2: only second column

Golden ratio mixing

Datta,Ling,Ramond hep-ph/0306002

$$s_{13}^2 = 0,\ s_{23}^2 = \frac{1}{2}$$

$$\begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ -\frac{1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & -\frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

with $\varphi = \frac{1+\sqrt{5}}{2}$

$\mathrm{GR1} : t_{12} = \dfrac{1}{\varphi}$ Datta,Ling,Ramond hep-ph/0306002

$\mathrm{GR2} : \theta_{12} = \dfrac{\pi}{5}$ Rodejohann 0810.5239

$\mathrm{GR3} : c_{12} = \dfrac{\varphi}{\sqrt{3}}$ Lam 1104.0055

Prediction of A_5, ...

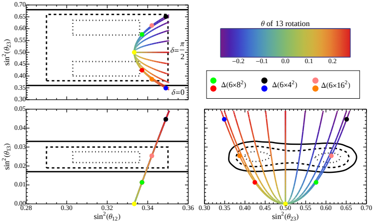
Common feature of popular mixing patterns: $\sin^2\theta_{13} = 0$ and $\sin^2\theta_{23} = \frac{1}{2}$.

Other mixing patterns possible as well:
e.g. HEXagonal $\theta_{12} = \frac{\pi}{6}$ [e.g. Dihedral group D_{12}] Albright,Dueck,Rodejohann 1004.2798

Top-down: predictions of different symmetry groups

For $G_\nu = Z_2 \times Z_2$

- $G_\ell = Z_3$: all groups $|G_f| < 1536$ (and $\Delta(1536)$) on parabola $\delta = 0$
- Abelian $G_\ell : 3 < |G_\ell| \leq 511$ new patterns



Holthausen, Lim, Lindner 1212.2411

Three successful groups in scan

$$\Delta(6 \times 10^2)[5], \quad \Delta(6 \times 16^2)[16],$$

$$(Z_{18} \times Z_6) \rtimes S_3[9] \equiv \langle T_3, S_3, U_3(9)\rangle < \Delta(6 \times 18^2)$$

$G_\nu = Z_2 \rightarrow$ trimaximal mixing

TM2: $U_{PMNS} = U_{tbm} U_{13}(\theta, \delta)$

possibly smaller groups interesting:

e.g. U_3 generator only contained in $\mathrm{Aut}(A_4)$ and not in A_4 itself

$$U_3(n) \equiv -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & z \\ 0 & z^* & 0 \end{pmatrix} \qquad \text{with } z^n = 1$$

Many groups have been studied. See e.g.  S_3 Fritzsch (1977);  S_4 Pakvasa,Sugiwara (1979);  A_5 Everett,Stuart 0812.1057; T_7 Luhn,Nasri,Ramond 0706.2341; Hagedorn,MS,Smirnov 0811.2955;
$\Sigma(81)$ Ma hep-ph/0612022; Hagedorn,MS,Smirnov 0811.2955;
T' Frampton,Kephart hep-ph/9409330, $Z_5 \rtimes Z_4$ MS,Smirnov 1110.0874; $Q_8 \rtimes A_4$ Holthausen,MS 1211.6953;
$\Delta(3n^2)$ Luhn,Nasri,Ramond hep-th/0701188; $\Delta(6n^2)$ Escobar,Luhn 0809.0639; ...

Top-down: predictions of different symmetry groups

For $G_\nu = Z_2 \times Z_2$

- $G_\ell = Z_3$: all groups $|G_f| < 1536$ (and $\Delta(1536)$) on parabola $\delta = 0$
- Abelian G_ℓ : $3 < |G_\ell| \leq 511$ new patterns

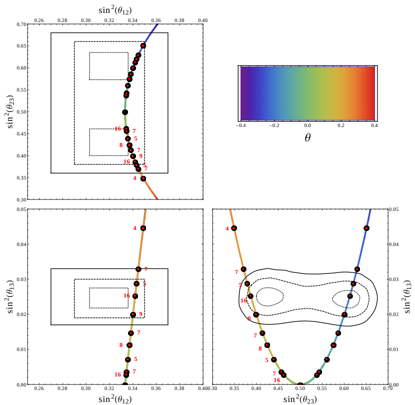
Three successful groups in scan

$$\Delta(6 \times 10^2)[5], \quad \Delta(6 \times 16^2)[16],$$

$$(Z_{18} \times Z_6) \rtimes S_3[9] \equiv \langle T_3, S_3, U_3(9) \rangle < \Delta(6 \times 18^2)$$

possibly smaller groups interesting:
e.g. U_3 generator only contained in $\mathrm{Aut}(A_4)$ and not in A_4 itself

$$U_3(n) \equiv - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & z \\ 0 & z^* & 0 \end{pmatrix} \qquad \text{with } z^n = 1$$



Holthausen, Lim, Lindner 1212.2411

$G_\nu = Z_2 \to$ trimaximal mixing TM2: $U_{PMNS} = U_{tbm} U_{13}(\theta, \delta)$

Many groups have been studied. See e.g.  S_3 Fritzsch (1977);  S_4 Pakvasa,Sugiwara (1979);
 A_5 Everett,Stuart 0812.1057; T_7 Luhn,Nasri,Ramond 0706.2341; Hagedorn,MS,Smirnov 0811.2955;
$\Sigma(81)$ Ma hep-ph/0612022; Hagedorn,MS,Smirnov 0811.2955;
T' Frampton,Kephart hep-ph/9409330, $Z_5 \rtimes Z_4$ MS,Smirnov 1110.0874; $Q_8 \rtimes A_4$ Holthausen,MS 1211.6953;
$\Delta(3n^2)$ Luhn,Nasri,Ramond hep-th/0701188; $\Delta(6n^2)$ Escobar,Luhn 0809.0639; . . .

Top-down: predictions of different symmetry groups

For $G_\nu = Z_2 \times Z_2$

- $G_\ell = Z_3$: all groups $|G_f| < 1536$ (and $\Delta(1536)$) on parabola $\delta = 0$
- Abelian G_ℓ : $3 < |G_\ell| \leq 511$ new patterns

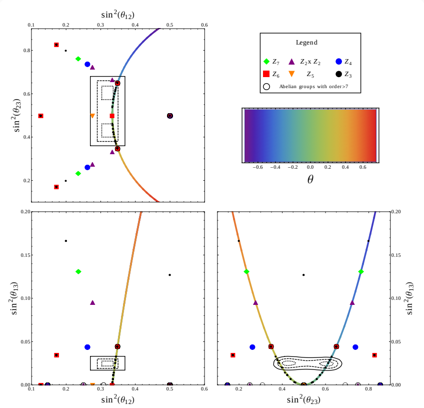
Three successful groups in scan

$$\Delta(6 \times 10^2)[5], \quad \Delta(6 \times 16^2)[16],$$

$$(Z_{18} \times Z_6) \rtimes S_3[9] \equiv \langle T_3, S_3, U_3(9) \rangle < \Delta(6 \times 18^2)$$

possibly smaller groups interesting:
e.g. U_3 generator only contained in $\mathrm{Aut}(A_4)$ and not in A_4 itself

$$U_3(n) \equiv -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & z \\ 0 & z^* & 0 \end{pmatrix} \qquad \text{with } z^n = 1$$



Holthausen, Lim, Lindner 1212.2411

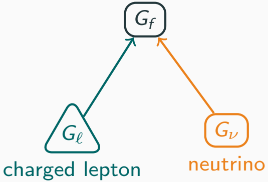
$G_\nu = Z_2 \rightarrow$ trimaximal mixing TM2: $U_{PMNS} = U_{tbm} U_{13}(\theta, \delta)$

Many groups have been studied. See e.g.  S_3 Fritzsch (1977);  S_4 Pakvasa,Sugiwara (1979);
 A_5 Everett,Stuart 0812.1057; T_7 Luhn,Nasri,Ramond 0706.2341; Hagedorn,MS,Smirnov 0811.2955;
$\Sigma(81)$ Ma hep-ph/0612022; Hagedorn,MS,Smirnov 0811.2955;
T' Frampton,Kephart hep-ph/9409330, $Z_5 \rtimes Z_4$ MS,Smirnov 1110.0874; $Q_8 \rtimes A_4$ Holthausen,MS 1211.6953;
$\Delta(3n^2)$ Luhn,Nasri,Ramond hep-th/0701188; $\Delta(6n^2)$ Escobar,Luhn 0809.0639; ...

Bottom-up approach

Reconstruct flavour symmetry from residual symmetry groups G_ℓ and G_ν Lam 0809.1185, 0907.2206

minimal symmetry of TBM is S_4



$$T = \begin{pmatrix} e^{\frac{2\pi i}{m}k_1} & & \\ & e^{\frac{2\pi i}{m}k_2} & \\ & & e^{\frac{2\pi i}{m}k_3} \end{pmatrix} \qquad S_1 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

and cyclic

maximal symmetry $G_\ell = U(1)^2$ and $G_\nu = Z_2 \times Z_2$

- direct model $G_\nu = Z_2 \times Z_2$
- semi-direct model $G_\nu = Z_2$
  continuous parameter Ge,Dicus,Repko 1104.0602,1108.0964
  Shimizu,Tanimoto,Watanabe 1105.2929; King,Luhn 1107.5332
  T broken [charged lepton corrections, solar sum rule]
  U broken, not S/SU [TM1/2, atmospheric sum rule]
- indirect model $G_{\ell,\nu} = \{1\}$ King 1304.6264, 1305.4846

complete classification for $G_\nu = Z_2 \times Z_2$ Fonseca,Grimus 1405.3678

von Dyck groups

Hernandez,Smirnov 1204.0445, 1212.2149

1. One-generator subgroups
   $G_\nu = \langle S_i \rangle = Z_2,\ S_i^2 = 1$ and
   $G_\ell = \langle T \rangle = Z_m,\ T^m = 1$
2. Construct von Dyck group
   $G_f = D(2, m, p)$ with
   $$W = U_{PMNS} S_i U_{PMNS}^\dagger T$$
   with $W^p = 1$

Group $D(n, m, p)$ is finite if

$$\frac{1}{n} + \frac{1}{m} + \frac{1}{p} > 1$$

thus there are only four finite ones

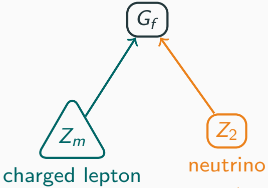
$$D_n \equiv D(2,2,n) \qquad A_4 \equiv D(2,3,3)$$
$$S_4 \equiv D(2,3,4) \qquad A_5 \equiv D(2,3,5)$$

Extensions: $G_\nu = Z_2 \times Z_2$ Hernandez,Smirnov 1212.2149; degenerate neutrino masses Hernandez,Smirnov 1304.7738

Bottom-up approach

Reconstruct flavour symmetry from residual symmetry groups G_ℓ and G_ν Lam 0809.1185, 0907.2206

minimal symmetry of TBM is S_4



$$T = \begin{pmatrix} e^{\frac{2\pi i}{m}k_1} & & \\ & e^{\frac{2\pi i}{m}k_2} & \\ & & e^{\frac{2\pi i}{m}k_3} \end{pmatrix} \qquad S_1 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

and cyclic

maximal symmetry $G_\ell = U(1)^2$ and $G_\nu = Z_2 \times Z_2$

- direct model $G_\nu = Z_2 \times Z_2$
- semi-direct model $G_\nu = Z_2$
  continuous parameter Ge,Dicus,Repko 1104.0602,1108.0964
  Shimizu,Tanimoto,Watanabe 1105.2929; King,Luhn 1107.5332
  T broken [charged lepton corrections, solar sum rule]
  U broken, not S/SU [TM1/2, atmospheric sum rule]
- indirect model $G_{\ell,\nu} = \{\mathbf{1}\}$ King 1304.6264, 1305.4846

complete classification for $G_\nu = Z_2 \times Z_2$ Fonseca,Grimus 1405.3678

von Dyck groups

Hernandez,Smirnov 1204.0445, 1212.2149

1. One-generator subgroups
   $G_\nu = \langle S_i \rangle = Z_2,\ S_i^2 = 1$ and
   $G_\ell = \langle T \rangle = Z_m,\ T^m = 1$
2. Construct von Dyck group
   $G_f = D(2, m, p)$ with

   $$W = U_{PMNS} S_i U_{PMNS}^\dagger T$$

   with $W^p = 1$

Group $D(n, m, p)$ is finite if

$$\frac{1}{n} + \frac{1}{m} + \frac{1}{p} > 1$$

thus there are only four finite ones

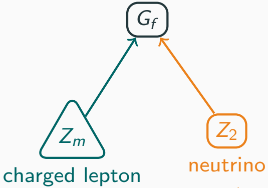
$$D_n \equiv D(2,2,n) \qquad A_4 \equiv D(2,3,3)$$
$$S_4 \equiv D(2,3,4) \qquad A_5 \equiv D(2,3,5)$$

Extensions: $G_\nu = Z_2 \times Z_2$ Hernandez,Smirnov 1212.2149;
degenerate neutrino masses Hernandez,Smirnov 1304.7738

Bottom-up approach

Reconstruct flavour symmetry from residual symmetry groups G_ℓ and G_ν Lam 0809.1185, 0907.2206

minimal symmetry of TBM is S_4



$$T = \begin{pmatrix} e^{\frac{2\pi i}{m} k_1} & & \\ & e^{\frac{2\pi i}{m} k_2} & \\ & & e^{\frac{2\pi i}{m} k_3} \end{pmatrix} \qquad S_1 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

and cyclic

maximal symmetry $G_\ell = U(1)^2$ and $G_\nu = Z_2 \times Z_2$

- direct model $G_\nu = Z_2 \times Z_2$
- semi-direct model $G_\nu = Z_2$
  continuous parameter Ge,Dicus,Repko 1104.0602,1108.0964
  Shimizu,Tanimoto,Watanabe 1105.2929; King,Luhn 1107.5332
  T broken [charged lepton corrections, solar sum rule]
  U broken, not S/SU [TM1/2, atmospheric sum rule]
- indirect model $G_{\ell,\nu} = \{\mathbf{1}\}$ King 1304.6264, 1305.4846

complete classification for $G_\nu = Z_2 \times Z_2$ Fonseca,Grimus 1405.3678

von Dyck groups

Hernandez,Smirnov 1204.0445, 1212.2149

1. One-generator subgroups
   $G_\nu = \langle S_i \rangle = Z_2$, $S_i^2 = 1$ and
   $G_\ell = \langle T \rangle = Z_m$, $T^m = 1$
2. Construct von Dyck group
   $G_f = D(2, m, p)$ with

   $$W = U_{PMNS} S_i U_{PMNS}^\dagger T$$

   with $W^p = 1$

Group $D(n, m, p)$ is finite if

$$\frac{1}{n} + \frac{1}{m} + \frac{1}{p} > 1$$

thus there are only four finite ones

$$D_n \equiv D(2,2,n) \qquad A_4 \equiv D(2,3,3)$$
$$S_4 \equiv D(2,3,4) \qquad A_5 \equiv D(2,3,5)$$

Extensions: $G_\nu = Z_2 \times Z_2$ Hernandez,Smirnov 1212.2149; degenerate neutrino masses Hernandez,Smirnov 1304.7738

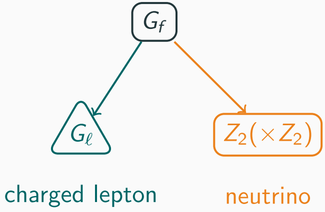
How to test flavour symmetries?

Precise predictions [$G_\nu = Z_2 \times Z_2$]

- Precise values of mixing parameters:

$\rightarrow$ For example tribimaximal mixing in models based on A_4, S_4

$$s_{12}^2 \equiv \frac{1}{3}, \quad s_{23}^2 \equiv \frac{1}{2}, \quad s_{13}^2 \equiv 0$$



Sum rules [$G_\nu = Z_2$]

- Relations between parameters:

$\rightarrow$ One example is an *atmospheric mixing sum rule*:

$$\sin\theta_{23} - \frac{1}{\sqrt{2}} = \lambda \sin\theta_{13}\cos\delta + \dots$$

- The most common models predict $\lambda \simeq 1$ or $\lambda \simeq -\frac{1}{2}$

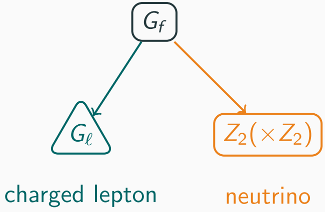
How to test flavour symmetries?

Precise predictions [$G_\nu = Z_2 \times Z_2$]

- Precise values of mixing parameters:

$\rightarrow$ For example tribimaximal mixing in models based on A_4, S_4

$$s^2_{12} \equiv \frac{1}{3}, \quad s^2_{23} \equiv \frac{1}{2}, \quad s^2_{13} \equiv 0$$



Sum rules [$G_\nu = Z_2$]

- Relations between parameters:

$\rightarrow$ One example is an *atmospheric mixing sum rule*:

$$\sin\theta_{23} - \frac{1}{\sqrt{2}} = \lambda \sin\theta_{13} \cos\delta + \dots$$

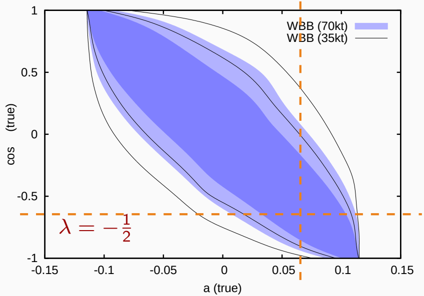
- The most common models predict $\lambda \simeq 1$ or $\lambda \simeq -\frac{1}{2}$

Testable predictons: sum rules

Atmospheric Sum Rule

Ballett, King, Luhn, Pascoli, MS 1308.4314

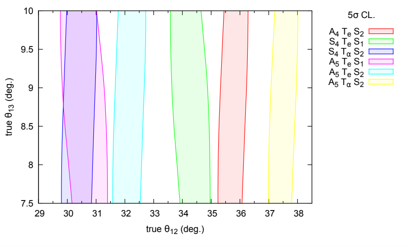
$$\frac{a}{\sqrt{2}} \equiv \sin\theta_{23} - \frac{1}{\sqrt{2}} \approx -\frac{1}{2}\sin\theta_{13}\cos\delta$$



- 2σ and 3σ allowed regions
- $\lambda \approx -\frac{1}{2}$: $A_4/S_4/A_5$ ($G_\nu = \langle S_2 \rangle$)
- Similar result for $\lambda = 1$: S_4/A_5 ($G_\nu = \langle S_1 \rangle$)

Solar Sum Rule

Ballett, King, Luhn, Pascoli, MS 1406.0308



- Solar sum rules, e.g. $A_4\ T_\alpha - S_2$

$$\sin\theta_{12} - \frac{1}{\sqrt{3}} = \sqrt{\frac{2}{2-\sin^2\theta_{13}}} - 1$$

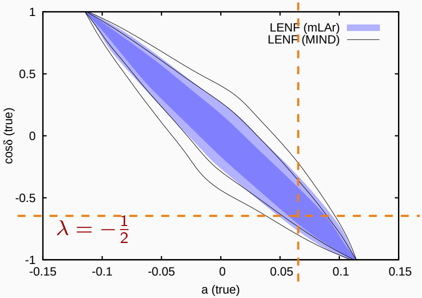
- 5σ allowed regions
- Clear separation of predictions
  Overlapping solar sum rules have different atmospheric sum rule
- Tested by JUNO-type experiment

Testable predictons: sum rules

Atmospheric Sum Rule

Ballett, King, Luhn, Pascoli, MS 1308.4314

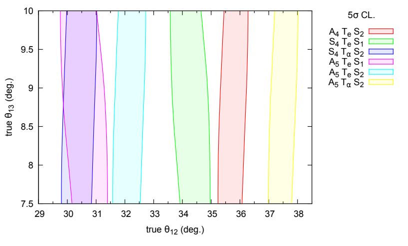
$$\frac{a}{\sqrt{2}} \equiv \sin\theta_{23} - \frac{1}{\sqrt{2}} \approx 1 \sin\theta_{13}\cos\delta$$



- 2σ and 3σ allowed regions
- $\lambda \approx -\frac{1}{2}$: $A_4/S_4/A_5$ $(G_\nu = \langle S_2 \rangle)$
- Similar result for $\lambda = 1$: S_4/A_5 $(G_\nu = \langle S_1 \rangle)$

Solar Sum Rule

Ballett, King, Luhn, Pascoli, MS 1406.0308



- Solar sum rules, e.g. $A_4\ T_\alpha - S_2$

$$\sin\theta_{12} - \frac{1}{\sqrt{3}} = \sqrt{\frac{2}{2-\sin^2\theta_{13}}} - 1$$

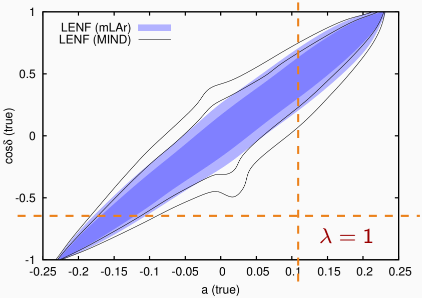
- 5σ allowed regions
- Clear separation of predictions
  Overlapping solar sum rules have different atmospheric sum rule
- Tested by JUNO-type experiment

Testable predictons: sum rules

Atmospheric Sum Rule

Ballett, King, Luhn, Pascoli, MS 1308.4314

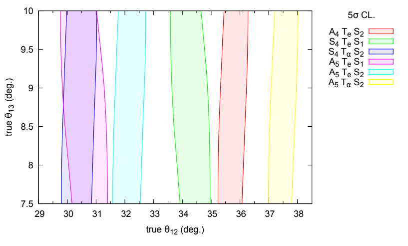
$$\frac{a}{\sqrt{2}} \equiv \sin\theta_{23} - \frac{1}{\sqrt{2}} \approx 1 \sin\theta_{13}\cos\delta$$



- 2σ and 3σ allowed regions
- $\lambda \approx -\frac{1}{2}$: $A_4/S_4/A_5$ $(G_\nu = \langle S_2 \rangle)$
- Similar result for $\lambda = 1$: S_4/A_5 $(G_\nu = \langle S_1 \rangle)$

Solar Sum Rule

Ballett, King, Luhn, Pascoli, MS 1406.0308



- Solar sum rules, e.g. $A_4\ T_\alpha - S_2$

$$\sin\theta_{12} - \frac{1}{\sqrt{3}} = \sqrt{\frac{2}{2-\sin^2\theta_{13}}} - 1$$

- 5σ allowed regions
- Clear separation of predictions
  Overlapping solar sum rules have different atmospheric sum rule
- Tested by JUNO-type experiment

Charged lepton corrections

- In GUTs typically $U_{\ell,\nu} \neq 1 \Rightarrow$ PMNS matrix $U = U_\ell^\dagger U_\nu$

  $U_{\ell,\nu} = P_{23} R_{23} R_{13} P_2 R_{12} P_{123}$ with $P_{ijk} = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ King hep-ph/0204360

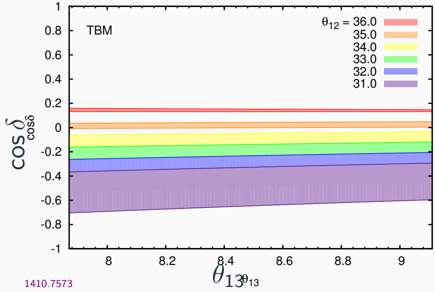
  e.g. quark-lepton complementarity: $U_\nu = U_{bm}$, $U_\ell = V_{CKM}$ MS,Smirnov hep-ph/0607232

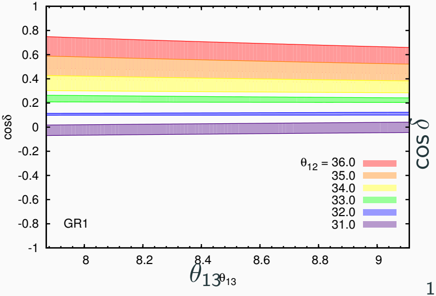
- If $\theta_{13}^\nu = \theta_{13}^\ell = 0$: $\tan\theta_{12}^\nu = \frac{|U_{\tau 1}|}{|U_{\tau 2}|}$ Ballett,King,Luhn,Pascoli,MS 1410.7573

$\Rightarrow$ Prediction for $\cos\delta$

$$\cos\delta = \frac{t_{23} s_{12}^2 + s_{13}^2 c_{12}^2 / t_{23} - s_{12}^{\nu 2}(t_{23} + s_{13}^2 / t_{23})}{2 s_{12} c_{12} s_{13}}$$

See for previous derivation Marzocca, Petcov, Romanino, Sevilla 1302.0423; Petcov 1405.6006





1410.7573

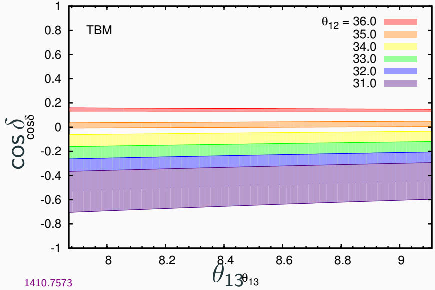
Charged lepton corrections

- In GUTs typically $U_{\ell,\nu} \neq 1 \Rightarrow$ PMNS matrix $U = U_\ell^\dagger U_\nu$
  $U_{\ell,\nu} = P_{23} R_{23} R_{13} P_2 R_{12} P_{123}$ with $P_{ijk} = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ King hep-ph/0204360
  e.g. quark-lepton complementarity: $U_\nu = U_{bm}$, $U_\ell = V_{CKM}$ MS,Smirnov hep-ph/0607232
- If $\theta_{13}^\nu = \theta_{13}^\ell = 0$: $\tan\theta_{12}^\nu = \frac{|U_{\tau 1}|}{|U_{\tau 2}|}$ Ballett,King,Luhn,Pascoli,MS 1410.7573

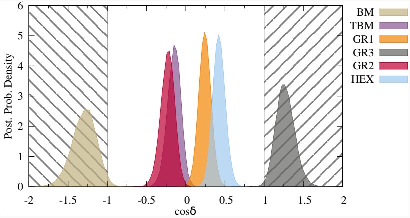
$\Rightarrow$ Prediction for $\cos\delta$

$$\cos\delta = \frac{t_{23} s_{12}^2 + s_{13}^2 c_{12}^2 / t_{23} - s_{12}^{\nu 2}(t_{23} + s_{13}^2 / t_{23})}{2 s_{12} c_{12} s_{13}}$$

See for previous derivation Marzocca, Petcov, Romanino, Sevilla 1302.0423; Petcov 1405.6006



1410.7573



BiMaximal; TriBiMaximal; Golden Ratio; HEXagonal

Charged lepton corrections

- In GUTs typically $U_{\ell,\nu} \neq 1 \Rightarrow$ PMNS matrix $U = U_\ell^\dagger U_\nu$

  $U_{\ell,\nu} = P_{23} R_{23} R_{13} P_2 R_{12} P_{123}$ with $P_{ijk} = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ King hep-ph/0204360

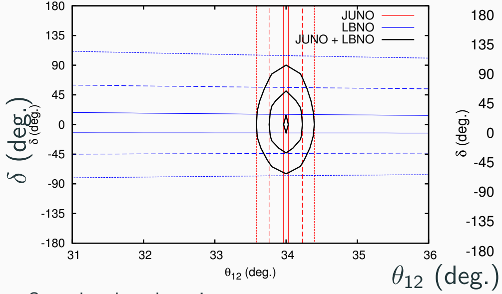
  e.g. quark-lepton complementarity: $U_\nu = U_{bm}$, $U_\ell = V_{CKM}$ MS,Smirnov hep-ph/0607232

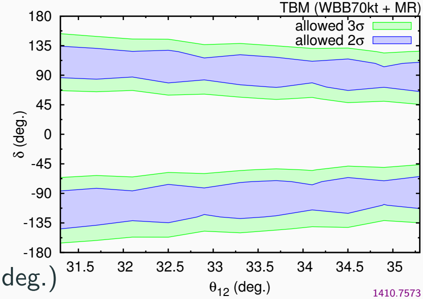
- If $\theta_{13}^\nu = \theta_{13}^\ell = 0$: $\tan\theta_{12}^\nu = \frac{|U_{\tau 1}|}{|U_{\tau 2}|}$ Ballett,King,Luhn,Pascoli,MS 1410.7573

$\Rightarrow$ Prediction for $\cos\delta$

$$\cos\delta = \frac{t_{23} s_{12}^2 + s_{13}^2 c_{12}^2 / t_{23} - s_{12}^{\nu 2}(t_{23} + s_{13}^2 / t_{23})}{2 s_{12} c_{12} s_{13}}$$

See for previous derivation Marzocca, Petcov, Romanino, Sevilla 1302.0423; Petcov 1405.6006





1410.7573

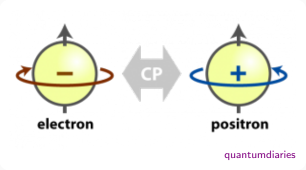
See related analyses in Girardi, Petcov, Titov 1410.8056, 1504.00658, 1504.02402, +Stuart 1509.02502

Flavour symmetries and CP

(Generalised) CP symmetry and flavour symmetry

(Generalised) CP symmetry

- Parity: reflection at origin $\vec{x} \to -\vec{x}$
- Charge conjugation: particle $\leftrightarrow$ anti-particle
- Generalised CP symmetry: $\phi \to X\phi^*$
  [with unitary transformation X in field space]



How to correctly use CP symmetry in presence of flavour symmetry?

CP symmetry is a "symmetry of the flavour symmetry"

[an element of the outer automorphism group] Holthausen,Lindner,MS 1211.5143

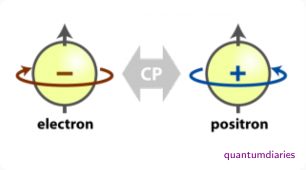
$$\begin{tikzcd}
\phi \arrow[r, "CP"] \arrow[rrr, bend right, "\text{flavour} \quad g'"'] & X\phi^* \arrow[r, "g", "\text{flavour}"'] & X\rho(g)^*\phi^* \arrow[r, "CP^{-1}"] & X\rho(g)^*X^{-1}\phi = \rho(g')\phi
\end{tikzcd}$$

- See also Feruglio, Hagedorn, Ziegler 1211.5560
- Physical CP symmetries are class-inverting Chen, Fallbacher, Mahanthappa, Ratz, Trautner 1402.0507
- Can provide solution to strong CP problem Antusch,Holthausen,MS,Spinrath 1307.0710

(Generalised) CP symmetry

- Parity: reflection at origin $\vec{x} \to -\vec{x}$
- Charge conjugation: particle $\leftrightarrow$ anti-particle
- Generalised CP symmetry: $\phi \to X\phi^*$

  [with unitary transformation X in field space]



How to correctly use CP symmetry in presence of flavour symmetry?

CP symmetry is a "symmetry of the flavour symmetry"

[an element of the outer automorphism group] Holthausen,Lindner,MS 1211.5143

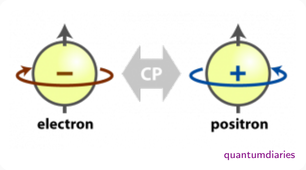
$$\begin{tikzcd} \phi \arrow[r, "CP"] \arrow[rrrr, bend right, "\text{flavour} \quad g'"'] & X\phi^* \arrow[r, "g", "\text{flavour}"'] & X\rho(g)^*\phi^* \arrow[r, "CP^{-1}"] & X\rho(g)^* X^{-1}\phi & \equiv \rho(g')\phi \end{tikzcd}$$

- See also Feruglio, Hagedorn, Ziegler 1211.5560
- Physical CP symmetries are class-inverting Chen, Fallbacher, Mahanthappa, Ratz, Trautner 1402.0507
- Can provide solution to strong CP problem Antusch,Holthausen,MS,Spinrath 1307.0710

(Generalised) CP symmetry and flavour symmetry

(Generalised) CP symmetry

- Parity: reflection at origin $\vec{x} \to -\vec{x}$
- Charge conjugation: particle $\leftrightarrow$ anti-particle
- Generalised CP symmetry: $\phi \to X\phi^*$
  [with unitary transformation X in field space]



How to correctly use CP symmetry in presence of flavour symmetry?

CP symmetry is a "symmetry of the flavour symmetry"

[an element of the outer automorphism group] Holthausen,Lindner,MS 1211.5143

$$\begin{tikzcd}
\phi \arrow[r, "CP"] \arrow[rrr, bend right, "\text{flavour} \quad g'"'] & X\phi^* \arrow[r, "g", "\text{flavour}"'] & X\rho(g)^*\phi^* \arrow[r, "CP^{-1}"] & X\rho(g)^* X^{-1}\phi \equiv \rho(g')\phi
\end{tikzcd}$$

- See also Feruglio, Hagedorn, Ziegler 1211.5560
- Physical CP symmetries are class-inverting Chen, Fallbacher, Mahanthappa, Ratz, Trautner 1402.0507
- Can provide solution to strong CP problem Antusch,Holthausen,MS,Spinrath 1307.0710

CP phases from partially breaking CP

Breaking to $G_\nu = Z_2 \times H^\nu_{\rm CP}$ Feruglio, Hagedorn, Ziegler 1211.5560,1303.7178

$[H^{(\nu)}_{\rm CP} = \{CP\}]$

$\boxed{G_f \rtimes H_{\rm CP}}$

G_ℓ $\boxed{Z_2 \times H^\nu_{\rm CP}}$

charged lepton

$Q^\dagger M_\ell^\dagger M_\ell Q = M_\ell^\dagger M_\ell$

$U_\ell^\dagger Q U_\ell = \mathrm{diag}$

neutrino

Z_2: $Z^T M_\nu Z = M_\nu$

CP: $X^T M_\nu X = M_\nu^*$

$X = \Omega_\nu \Omega_\nu^T$

$\Omega_\nu^\dagger Z \Omega_\nu = \mathrm{diag}$

$$U_{PMNS} = U_\ell^\dagger \Omega_\nu R(\theta) K_\nu$$

- $U_\ell \to U_\ell K_\ell$ removes 3 unphysical phases
- $\Omega_\nu^T M_\nu \Omega_\nu$ block diagonal $\to R(\theta)$
- $U_\nu \to U_\nu K_\nu$ for real positive ν masses
- U_{PMNS} def'd up to permut'n of rows/columns

Predictions

Mixing angles and all CP phases as function of one parameter θ up to exchange of rows and columns

Crucial ingredients

Flavour and CP symmetry and breaking pattern

Applied to many groups

S_4 Feruglio,Hagedorn,Ziegler 1211.5560,1303.7178

Ding,King,Luhn,Stuart 1303.6180 Ding,Zhou 1312.4401

Ding,Li 1312.4401,1408.0785

A_4 Ding,King,Stuart 1307.4212

A_5 Ding,Li 1503.03711 Di Iura, Hagedorn, Meloni 1503.04140

Ballett, Pascoli, Turner 1503.07543

$\Delta(3n^2), \Delta(6n^2)$ Hagedorn,Meroni,Molinaro 1408.7118

King,Neder 1403.1758 Ding, King, Neder 1409.8005

Ding,King 1403.5846; Ding,Zhou 1404.0592

CP phases from partially breaking CP

Breaking to $G_\nu = Z_2 \times H^\nu_{\mathrm{CP}}$ Feruglio, Hagedorn, Ziegler 1211.5560,1303.7178

$[H^{(\nu)}_{\mathrm{CP}} = \{CP\}]$

$\boxed{G_f \rtimes H_{\mathrm{CP}}}$

G_ℓ $\qquad\qquad$ $\boxed{Z_2 \times H^\nu_{\mathrm{CP}}}$

charged lepton

$Q^\dagger M_\ell^\dagger M_\ell Q = M_\ell^\dagger M_\ell$

$U_\ell^\dagger Q U_\ell = \mathrm{diag}$

neutrino

Z_2: $Z^T M_\nu Z = M_\nu$

CP: $X^T M_\nu X = M_\nu^*$

$X = \Omega_\nu \Omega_\nu^T$

$\Omega_\nu^\dagger Z \Omega_\nu = \mathrm{diag}$

$$U_{PMNS} = U_\ell^\dagger \Omega_\nu R(\theta) K_\nu$$

- $U_\ell \to U_\ell K_\ell$ removes 3 unphysical phases
- $\Omega_\nu^T M_\nu \Omega_\nu$ block diagonal $\to R(\theta)$
- $U_\nu \to U_\nu K_\nu$ for real positive ν masses
- U_{PMNS} def'd up to permut'n of rows/columns

Predictions

Mixing angles and all CP phases as function of one parameter θ up to exchange of rows and columns

Crucial ingredients

Flavour and CP symmetry and breaking pattern

Applied to many groups

S_4 Feruglio,Hagedorn,Ziegler 1211.5560,1303.7178

Ding,King,Luhn,Stuart 1303.6180 Ding,Zhou 1312.4401

Ding,Li 1312.4401,1408.0785

A_4 Ding,King,Stuart 1307.4212

A_5 Ding,Li 1503.03711 Di Iura, Hagedorn, Meloni 1503.04140

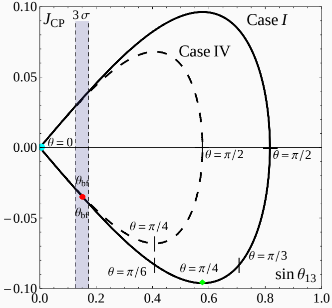
Ballett, Pascoli, Turner 1503.07543

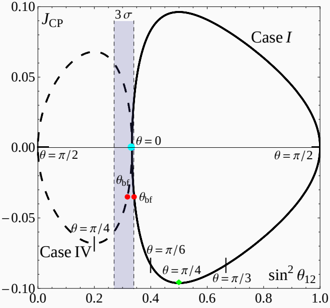
$\Delta(3n^2), \Delta(6n^2)$ Hagedorn,Meroni,Molinaro 1408.7118

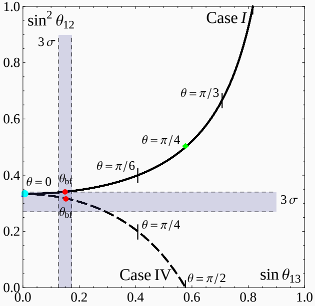
King,Neder 1403.1758 Ding, King, Neder 1409.8005

Ding,King 1403.5846; Ding,Zhou 1404.0592

$S_4 \rtimes H_{\mathrm{CP}}$







7 interesting cases identified Feruglio,Hagedorn,Ziegler 1211.5560

$G_\ell = Z_3$:

I/IV: $\sin\alpha_{ji} = 0$ and $|\sin\delta| = 1$ $[J_{CP}|_{bf} = 0.348]$

II/V: $\sin\alpha_{ji} = 0$ and $|\sin\delta| = 0$

III: $\sin\alpha_{ji} \neq 0$, but $\sin^2\theta_{23}\big|_{bf} = 0.349, 0.651$

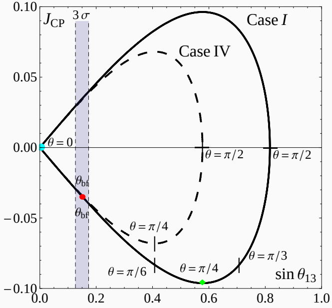
$G_\ell = Z_2 \times Z_2, Z_4$:

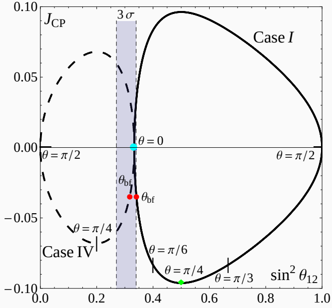
a/b: $\sin\alpha_{ji} = 0$ and $J_{CP} = 0$

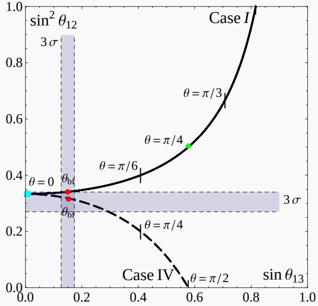
Similar results for $A_4 \rtimes H_{\mathrm{CP}}$: $|\sin\delta| = 0, 1$

Feruglio,Hagedorn,Ziegler 1211.5560; Ding,King,Stuart 1307.4212

$S_4 \rtimes H_{\rm CP}$







7 interesting cases identified Feruglio,Hagedorn,Ziegler 1211.5560

$G_\ell = Z_3$:

I/IV: $\sin\alpha_{ji} = 0$ and $|\sin\delta| = 1$ $[J_{CP}|_{bf} = 0.348]$

II/V: $\sin\alpha_{ji} = 0$ and $|\sin\delta| = 0$

III: $\sin\alpha_{ji} \neq 0$, but $\sin^2\theta_{23}\big|_{bf} = 0.349, 0.651$

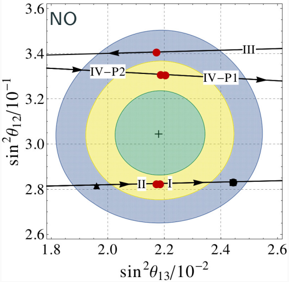
$G_\ell = Z_2 \times Z_2, Z_4$:

a/b: $\sin\alpha_{ji} = 0$ and $J_{CP} = 0$

Similar results for $A_4 \rtimes H_{\rm CP}$: $|\sin\delta| = 0, 1$

Feruglio,Hagedorn,Ziegler 1211.5560; Ding,King,Stuart 1307.4212

$A_5 \rtimes H_{\mathrm{CP}}$



Di Iura,Hagedorn,Meloni 1503.04140

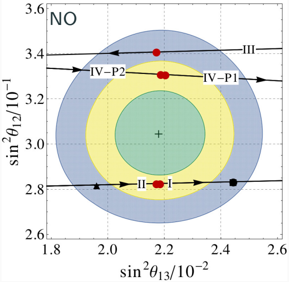
Li,Ding 1503.03711; Di Iura,Hagedorn Meloni 1503.04140; Ballett,Pascoli,Turner 1503.07543

Majorana phases $\sin\alpha_{ij} = 0$

Dirac phase

- $|\sin\delta| = 0$ for $G_\ell = Z_2 \times Z_2$
- $|\sin\delta| = 1$ for $G_\ell = Z_5$ (II) and $G_\ell = Z_3$ (III)

G_ℓ	θ_{12}	θ_{23}
Z_5 (II)	$31.72° + 12.52° s_{13}$	$45°$
Z_3 (III)	$35.27° + 14.33° s_{13}$	$45°$



Di Iura,Hagedorn,Meloni 1503.04140

Li,Ding 1503.03711; Di Iura,Hagedorn Meloni 1503.04140; Ballett,Pascoli,Turner 1503.07543

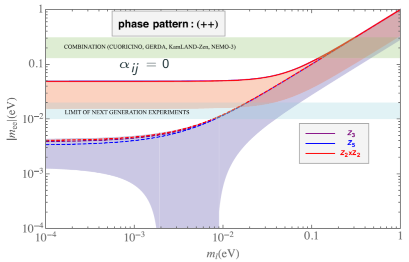
Majorana phases $\sin\alpha_{ij} = 0$

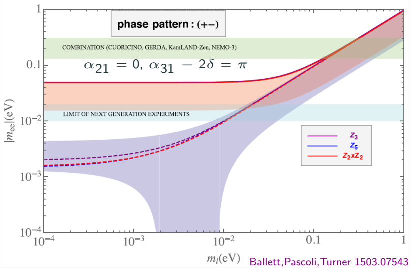
Dirac phase

- $|\sin\delta| = 0$ for $G_\ell = Z_2 \times Z_2$
- $|\sin\delta| = 1$ for $G_\ell = Z_5$ (II) and $G_\ell = Z_3$ (III)

G_ℓ	θ_{12}	θ_{23}
Z_5 (II)	$31.72^\circ + 12.52^\circ s_{13}$	45°
Z_3 (III)	$35.27^\circ + 14.33^\circ s_{13}$	45°

Implications for $\beta\beta0\nu$ Ding,Li 1503.03711; Ballett,Pascoli,Turner 1503.07543

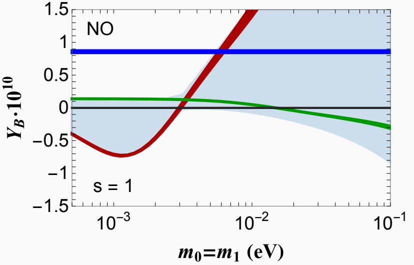




Ballett,Pascoli,Turner 1503.07543

$\Delta(3N^2) \rtimes H_{\rm CP}$ and $\Delta(6N^2) \rtimes H_{\rm CP}$

- $G_\nu = (Z_2 \times Z_2) \rtimes H_{\rm CP}$ [only $\Delta(6N^2)$] King,Neder 1403.1758
  TM2 $|\sin\delta| = 0, 1$, $\alpha_{31} = \mathbf{Z}\pi$, $\alpha_{21} \in \mathbf{Q}$: discrete param's, lower b'nd $|m_{ee}|$
- $G_\nu = Z_2 \times H_{\rm CP}$ Hagedorn,Meroni,Molinaro 1408.7118; Ding,King,Neder 1409.8005; Hagedorn,Molinaro 1602.04206
  Depends on symmetry breaking: continuous θ and discrete parameters

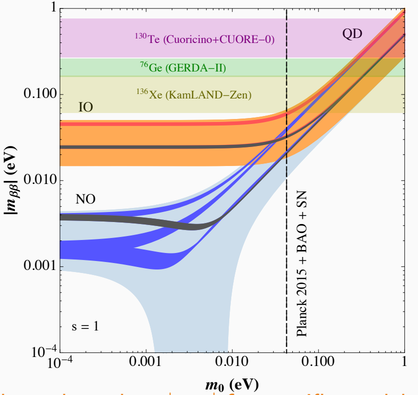


Hagedorn,Molinaro 1602.04206
Hagedorn,Mohapatra,Molinaro,Nishi,Petcov 1711.02866

$N = 8$ and $m = 4$

dark-gray $k_1 = k_2 = 0 \rightarrow$

$\uparrow$ red $\zeta = 0, \pi$, green $\zeta = \frac{\pi}{2}, \frac{3\pi}{2}$, light-blue $\zeta \in \mathcal{I}_i$

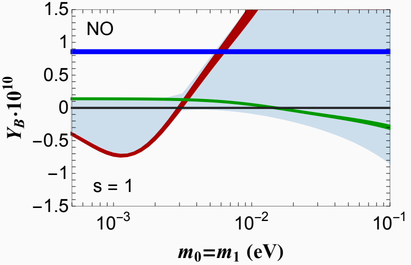


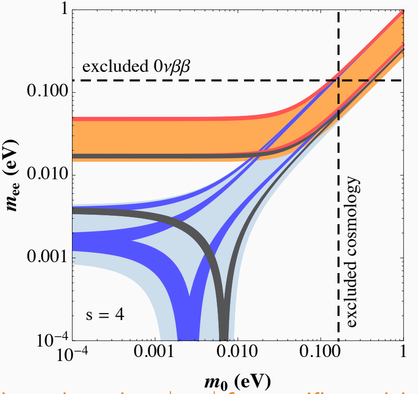
lower bound on $|m_{ee}|$ for specific models
however no general lower bound

See also detailed studies of $\Delta(96)$ Ding,King 1403.5846 and $\Delta(48)$ Ding,Zhou 1404.0592

$\Delta(3N^2) \rtimes H_{\mathrm{CP}}$ and $\Delta(6N^2) \rtimes H_{\mathrm{CP}}$

- $G_\nu = (Z_2 \times Z_2) \rtimes H_{\mathrm{CP}}$ [only $\Delta(6N^2)$] King,Neder 1403.1758
  TM2 $|\sin\delta| = 0, 1$, $\alpha_{31} = \mathbf{Z}\pi$, $\alpha_{21} \in \mathbf{Q}$: discrete param's, lower b'nd $|m_{ee}|$
- $G_\nu = Z_2 \times H_{\mathrm{CP}}$ Hagedorn,Meroni,Molinaro 1408.7118; Ding,King,Neder 1409.8005; Hagedorn,Molinaro 1602.04206
  Depends on symmetry breaking: continuous θ and discrete parameters





Hagedorn,Molinaro 1602.04206
Hagedorn,Mohapatra,Molinaro,Nishi,Petcov 1711.02866

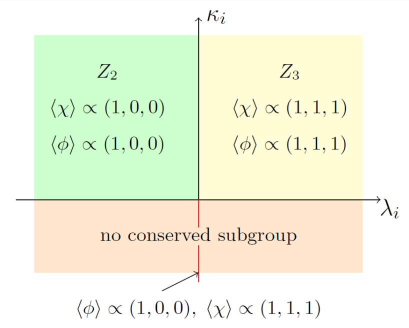
$N = 8$ and $m = 4$
dark-gray $k_1 = k_2 = 0 \rightarrow$
$\uparrow$ red $\zeta = 0, \pi$, green $\zeta = \frac{\pi}{2}, \frac{3\pi}{2}$, light-blue $\zeta \in \mathcal{I}_i$

lower bound on $|m_{ee}|$ for specific models
however no general lower bound

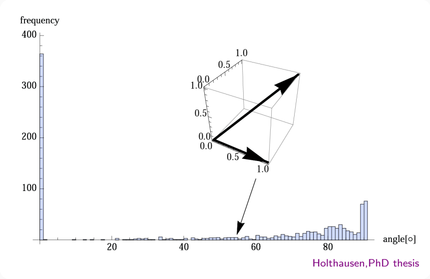
See also detailed studies of $\Delta(96)$ Ding,King 1403.5846 and $\Delta(48)$ Ding,Zhou 1404.0592

Symmetry breaking

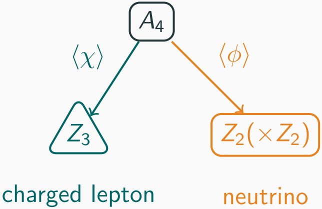
Flavour symmetry breaking: example A_4



Holthausen,MS 1111.1730



- Generally zero or random mixing due to term $[\chi^2]_{1'}[\phi^2]_{1''}$
- Enhanced symmetry $G_f \times G_f$ in scalar potential required for correct symmetry breaking pattern

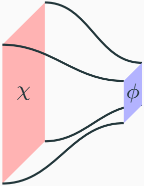


Enhanced symmetry in scalar potential

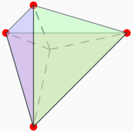
Extra-dimensions

Brane-localized flavon fields ϕ, χ
$\rightarrow$ enhanced symmetry $G_f \times G_f$

e.g. He,Keum,Volkas hep-ph/0601001



Explicit breaking via boundary conditions on orbifold



e.g. T^2/Z_2^3

Kobayashi,Omura,Yoshioka 0809.3064

Driving fields in SUSY Altarelli,Feruglio hep-ph/0512103

- Introduce $U(1)_R$ symmetry to constrain flavon potential
- Driving fields with R-charge 2 impose constraint equations

Example A_4: $\varphi \sim 3_0$, $\phi \sim 3_2$

$$W = \phi(M\varphi + g\varphi\varphi) \quad \Rightarrow \qquad 0 \overset{!}{=} \frac{\partial W}{\partial \phi_1} = M\varphi_1 + \frac{2g}{3}(\varphi_1^2 - \varphi_2\varphi_3) \quad \text{and cyclic}$$

leads to $\langle\varphi\rangle \sim (1,0,0)^T$

Enhanced symmetry in scalar potential: group extension

The existence of a short exact sequence Holthausen,MS 1111.1730

$$1 \longrightarrow \mathrm{N} \xrightarrow{\alpha} \mathrm{G} \xrightarrow{\beta} \mathrm{H} \longrightarrow 1$$ with α injective and β surjective

implies $G/N \cong H$ and every rep'n ρ^H of H there is rep'n $\rho = \rho^H \circ \beta$ of G

- Induced rep'n ρ has same properties as ρ^H

  [e.g. $T'/Z_2 \cong A_4$: same predictions of T' and A_4 if only singlet and triplet rep'n Feruglio,Hagedorn,Lin,Merlo hep-ph/0702194]
- Extension can lead to enhanced symmetry $G \times H$ in scalar potential

Examples are semi-direct products ($G = N \rtimes H$):

In $G = Q_8 \rtimes A_4$: $\chi \sim 3_{A_4}$ and $\phi_i \sim 4_G$ Holthausen,MS 1111.1730; Holthausen,Lindner,MS 1211.5143

By construction product $(\phi_i\phi_i)_{1'} = 0$ is antisymmetric and thus vanishes

$\Rightarrow$ Enhanced $[(Q_8 \rtimes A_4) \times A_4] \times Z_4$ symmetry in scalar potential, but explicitly broken to $(Q_8 \rtimes A_4) \times Z_4$ by Yukawa couplings

- No supersymmetry or extra dimensions

$\Rightarrow$ Dynamical low-scale flavour symmetry breaking possible

Conclusions

Conclusions

flavour (and CP) symmetries are predictive

measurement of $\theta_{13} \neq 0$ excluded models
$\rightarrow$ more intricate models required

measurement of Dirac CP phase important

helps to test flavour symmetry
and distinguish between different groups

open theoretical questions

symmetry breaking
fermion masses

Thank you!

Conclusions

flavour (and CP) symmetries are predictive

measurement of $\theta_{13} \neq 0$ excluded models
$\rightarrow$ more intricate models required

measurement of Dirac CP phase important

helps to test flavour symmetry
and distinguish between different groups

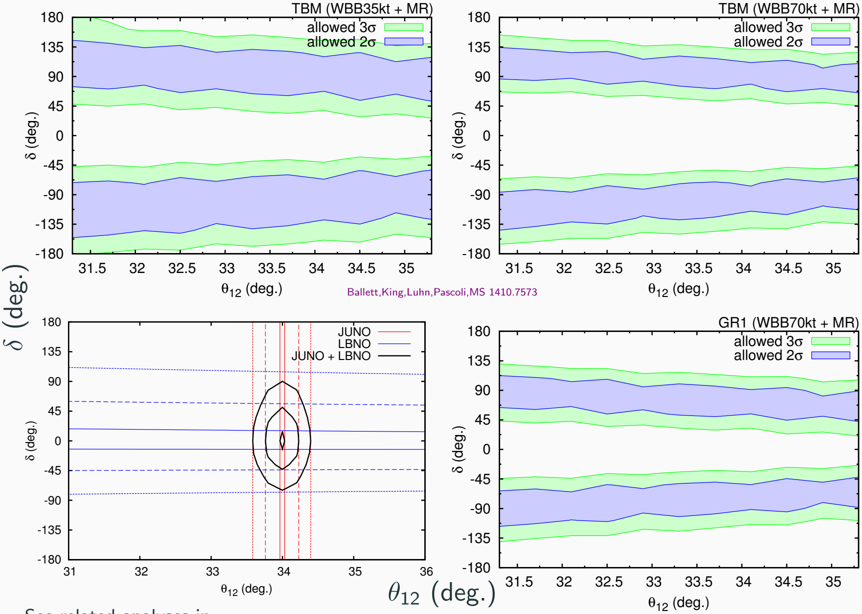
open theoretical questions

symmetry breaking
fermion masses

Thank you!

Backup slides

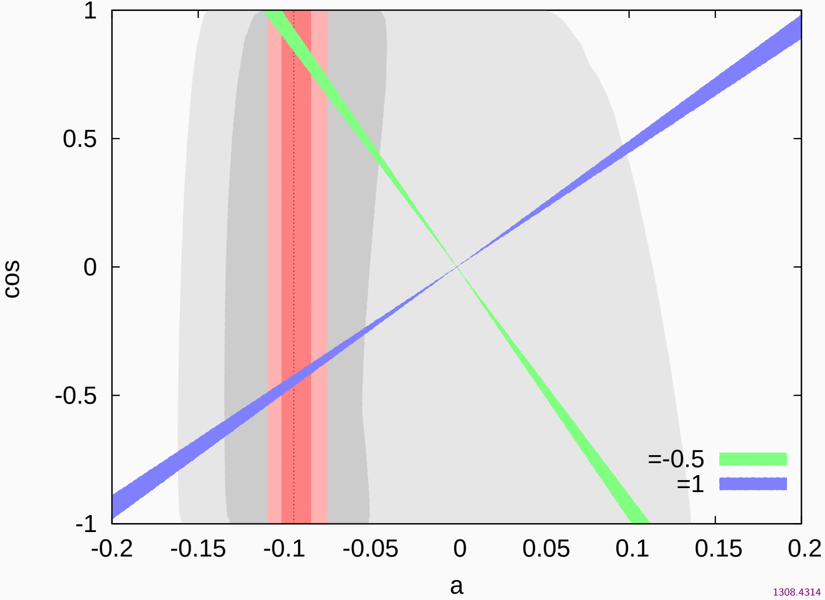
Charged lepton corrections: experimental test



Ballett,King,Luhn,Pascoli,MS 1410.7573

See related analyses in Girardi, Petcov, Titov 1410.8056, 1504.00658, 1504.02402, +Stuart 1509.02502

Atmospheric sum rule



1308.4314

Atmospheric sum rule: fitting